Macroscopic Einstein equations for a system of interacting particles and their cosmological applications.

A. V. Zakharov

Dept.Gen.Rel.& Grav.
Kremljevskaja Str. 18
Kazan Stata University
420008 Kazan, Russia
tel.:(+7)(8432)315586
e-mail: Alexei.Zakharov@ksu.ru

SUMMARY

One of the possible applications of macroscopic Einstein equations has been considered. So, the nonsingular isotropic and uniform cosmological model is built. The cosmological consequences of this model are agree with conclusions of standard hot model of the Universe.

Macroscopic Einstein equations for a system of interacting particles and their cosmological applications.

A. V. Zakharov

1. Macroscopic system of Einstein equations

As it is known the macroscopic Maxwell equation for continuous media can be obtained also from the microscopic Maxwell equations by ensemble averaging the latter (refer to.[1]-[3]).

The Einstein equations, which right-hand sides contain the energy-momentum tensor of matter, are phenomenological equations. It is natural to suppose that the Einstein equations (or their generalizations) for continuous media can be also obtained from the microscopic Einstein equations, i.e., Einstein equations which right-hand side contain the sum of the energy - momentum tensors of individual particles. However, due to the nonlinearity of the Einstein equations left-hand side, the averaging of the microscopic Einstein equations is more complicated than one of the microscopic Maxwell equations (refer to. [4] - [6]).

In Ref. [7] - [9] were obtained the macroscopic Einstein equations for a system of interaction particles up to second order in the interaction constant. They have the forms:

$$G_{ij} + Z_{ij} = \chi T_{ij}, \tag{1}$$

Here G_{ij} is the Einstein's tensor of the Riamannian space with macroscopic metric g_{ij}, T_{ij} is the macroscopic energy - momentum tensor, $\chi = 8\pi k/c^4$ is Einstein constant, k is the gravitational constant, c is the velosity of light.

The Einstein equations of the gravitational field for continium media differ from the classical Einstein equations by the presence of additional tensor Z_{ij} in the left-hand side. It caused by particle interaction.

The form of this tensor is given in Ref.[8] in the case of gravitationally interaction of particles and in Ref.[9] in the case of electromagnetically interaction of particles.

The tensor Z_{ij} is the traceless tensors with zero divergence:

$$g^{ij} Z_{ij} = 0, \qquad Z^{ij}_{;j} = 0. \tag{2}$$

where the semicolon denote a covariant derivative in a spase with the macroscopic metric g_{ij}.

In the case of gravitationally interacting particles the tensor Z_{ij} has the form (Ref. [8]):

$$Z_{ij} = \varphi^{k}_{ij;k} + \mu_{ij}, \tag{3}$$

where

$$\varphi_{ij}^{k} = -\sum_{bc} \frac{\chi^3 m_b^3 m_c^3 c^9}{8(2\pi)^3} \int \frac{d^3p'}{p'^0\sqrt{(-g)}} \int \frac{d^3p''}{p''^0\sqrt{(-g)}} \Big[\frac{1}{2} g^{fk} u_i'' u_j'' +$$

$$+ u'^k (u'u'')(\delta_j^f u_i'' + \delta_i^f u_j'')\Big] \left((u'u'')^2 - \frac{1}{2}\right)$$

$$K_{f\alpha}(u', u'') \left(F_c'' \frac{\partial F_b'}{\partial p_\alpha'} - F_b' \frac{\partial F_c''}{\partial p_\alpha''}\right), \tag{4}$$

$$\mu_{ij} = -\sum_{bc} \frac{\chi^3 m_b^3 m_c^3 c^9}{8(2\pi)^3} \int \frac{d^3p'}{p'^0\sqrt{(-g)}} \int \frac{d^3p''}{p''^0\sqrt{(-g)}} \Big\{ \Big[(z^2 + \frac{1}{2})(u_i'' u_j'' + u_i' u_j') +$$

$$+ (z^2 - \frac{1}{2}) g_{ij} - 2z(u_i' u_j'' + u_i'' u_j')\Big] g^{qr} - 2(z^2 - \frac{1}{2}) \delta_i^q \delta_j^r \Big\}$$

$$F_c'' \frac{\partial}{\partial p_f'} \Big\{ F_b' \Big[(z^2 - \frac{1}{2}) \delta_f^m + (z^2 + \frac{1}{2}) u_f' u'^m - 2z u_f'' u'^m \Big] \Big\} J_{rqm}(u', u''). \tag{5}$$

Here p'^i is the momentum of particles of species b, p''^i is the momentum of particles of spesies c $u'^i = p'^i/m_b c$, $u''^i = p''^i/m_c c$, F_b' is the one - particle distribution function of particles of spesies b, F_c'' is the one - particle distribution function of spesies c, $z = u'^i u_i''$, m_b and m_c are the mass of particles of spesies b and c respectively,

$$\frac{d^3p'}{p'^0\sqrt{(-g)}} \qquad and \qquad \frac{d^3p''}{p''^0\sqrt{(-g)}}$$

are the invariant volume elements in tree - dimensional momentum space of particles spesies "b" and "c" respectively.

The greek index α in (4) takes the values 1,2 and 3 only (the spartial index). The derivative with respect to p_f' in (5) should be calculated as all four components of momentum are independent. The dependence of p_0' on p_α' is taken into account after differentiation with respect p_f' is completed.

The tensors $K_{ij}(u', u'')$ and $J_{ijk}(u', u'')$ have the form:

$$K_{ij}(u', u'') = \frac{4\pi^2}{k_{min}^2[(u'u'')^2 - 1]^{3/2}} \{ -[(u'u'')^2 - 1] g_{ij} -$$

$$- u_i' u_j' - u_i'' u_j'' + (u'u'')(u_i' u_j'' + u_i'' u_j')\} \tag{6}$$

$$J_{ijk}(u', u'') = A\Big[(g_{ij} u_k' + g_{ik} u_j' + g_{jk} u_i') - z(g_{ij} u_k'' + g_{ik} u_j'' + g_{jk} u_i'') -$$

$$-(u_i' u_j'' u_k'' + u_i'' u_j' u_k'' + u_i'' u_j'' u_k') + 3z u_i'' u_j'' u_k''\Big] +$$

$$+ C\Big[u_i' u_j' u_k' - z(u_i' u_j' u_k'' + u_i' u_j'' u_k' + u_i'' u_j' u_k') +$$

$$+ z^2(u_i' u_j'' u_k'' + u_i'' u_j' u_k'' + u_i'' u_j'' u_k') - z^3 u_i'' u_j'' u_k''\Big], \tag{7}$$

where

$$A = -\frac{2\pi\sqrt{2}}{k_{min}}\Big[\frac{(z-2)}{(z-1)^2(z+1)^{1/2}} + \frac{(2z-1)}{(z+1)(z-1)^{5/2}}\ln\left(z+\sqrt{z^2-1}\right)\Big], \tag{8}$$

$$C = -\frac{2\pi\sqrt{2}}{k_{min}}\Big[\frac{(z-6)}{(z-1)^3(z+1)^{3/2}} + \frac{(6z-1)}{(z+1)^2(z-1)^{7/2}}\ln\left(z+\sqrt{z^2-1}\right)\Big]. \tag{9}$$

In these expressions

$$k_{min} = \frac{1}{r_{max}},$$

where r_{max} is the size of the correlation region in the case of gravitationally interacting particles. In Refs. [10], [11] there are estimates for r_{max} in the case where the average metric g_{ij} is the metric of isotropic cosmological model.

2. Applications of the theory. Nonsingular isotropic and uniform cosmological models in macroscopic theory of gravity.

Let's consider received equations for the ambience, base at local thermodynamic balance. In this cause the distribution function of each sorts of particles has the type ((refer to. [12]):

$$F_a(p_\alpha) = A_a exp[-c(v_i p^i)/(k_B T)]. \tag{10}$$

Here v_i is the macroscopic four-velocities of ambience, k_B is the Boltzman's constant, T is the temperature, A_a is the normalizing constant.

In this case tensor φ^k_{ij} becomes zero, but tensor μ_{ij} has the form:

$$\mu_{ij} = \chi\epsilon_1\left(\frac{4}{3}v_i v_j - \frac{1}{3}g_{ij}\right) \tag{11}$$

This is correct both for gravitational and electromagnetic interactions.

If move μ_{ij} from left-hand side to the right-hand side , the macroscopic equation effectively change into usual Einstein equations with the additional tensor of energy-momentum in the right-hand side. Moreover the "additional tensor of energy-momentum" has the form of energy-momentum tensor of ideal liquid with the state equation $P_1 = \epsilon_1/3$, but with negative "density of energy" $(-\epsilon_1)$ and negative "pressure" $(-P_1)$. Via ϵ_1 and P_1 their absolute values are marked.

In the case of gravitational interactions in nonrelativistic limit when $mc^2 >> k_B T$, the absolute value of this "negative density of energy" is

$$\epsilon_1 = \sum_{ab}\frac{4k^2 r_{max}}{k_B T}m_a^2 m_b^2 N_a N_b, \tag{12}$$

where N_a and N_b is the number of particles of species a and b recpectively densities, m_a and m_b are their masses.

It is proportional to the square of gravitational constant and square of number of particles density. Consequently the additional terms can play significant role in ambiences of sufficiently high density. Such density is possible on the early stages of the Universe evolution. So, naturally, one can use the first exhibits of the received equations in the theories of the early stages of the Universe evolution.

Thereby there is a real possibility of manifestation an interaction on early stages of the Universe evolution.

Let's turn to the building of uniform and isotropic cosmological models within the framework of macroscopic theory of gravity.

Let's write the metricses of these models in a form:

$$(ds)^2 = a^2(\eta)\left((d\eta)^2 - (dr)^2 - \phi^2(r)((d\theta)^2 + \sin^2(\theta)(d\varphi)^2)\right). \tag{13}$$

Here $\phi(r) = r$, $\phi(r) = \sin r$, $\phi(r) = \sinh r$ for the flat, close and open models respectively.

The system of Einstein equations for these metricses is reduced to two equations (refer to ,for instance, [13]):

$$a'^2 + \xi a^2 = \frac{8\pi k}{3c^4} a^4 \tilde{\epsilon}, \tag{14}$$

$$\frac{d\tilde{\epsilon}}{\tilde{\epsilon} + \tilde{P}} = -3\frac{da}{a}. \tag{15}$$

Here stroke under a marks derivative on time variable η; $\tilde{\epsilon}$ and $\tilde{P}$ is a density of energy and pressure of matter , $\xi = 0, +1, -1$ for the flat, closed and open models respectively.

The macroscopic Einstein equations are distinguish from (14) - (15) by following. In the first, $\tilde{\epsilon}$ in the right-hand side of (14) is replased by the difference of usual energy density ϵ and absolute value "density of energy" ϵ_1 , stipulated by the particles interaction of ambience.

At the present moment of the Universe evolution the expression (12) for ϵ_1 is correct.

In the second, the equation (15) is changed by two similar:

$$\frac{d\epsilon}{\epsilon + P} = -3\frac{da}{a}, \tag{16}$$

$$\frac{d\epsilon_1}{\epsilon_1 + P_1} = -3\frac{da}{a}. \tag{17}$$

Here P is a usual pressure of matter, $P_1 = \epsilon_1/3$ is the "pressure", stipulated by the interaction of particles.

Equations (16) and (17) follows from the fact, that divergency of the usual energy - momentum tensor and divergency of the "additional energy - momentum tensor", stipulated by the interaction, equals to zero.

From (12) it follow that in present the value ϵ_1 and the relict radiation energy density are of the same order of value (see to below). So, if we take into account a contribution ϵ_1 to $\tilde{\epsilon}$, we must take into account a contribution of the relict radiation energy density to $\tilde{\epsilon}$ density.

So, let's put in the right part of (14)

$$\tilde{\epsilon} = \epsilon_m - \bar{\epsilon}. \tag{18}$$

Here ϵ_m is the density of energy of material disregarding density of energy of relict radiation, but under $\bar{\epsilon}$ we shall understand a difference between the absolute value "density of energy", stipulated by the interaction, and density of energy of relict radiation. This is suitable, since equations of condition for relict radiation and for "energy - momentum tensor", stipulated by the interaction alike: pressure is one third from density of energy. So that the equation (16) and (17) coused from the following $\bar{\epsilon}$ dependency on the scale factor:

$$\bar{\epsilon} = \frac{3c^4 a_1^2}{8\pi k a^4}. \tag{19}$$

Here $a_1 = const$.

For density of energy of rest material from (16) we have:

$$\epsilon_m = \frac{3c^4 a_0}{4\pi k a^3}, \tag{20}$$

where $a_0 = const$.

We substitute (18), (19) and (20) into (14). As a result we obtain the equation:

$$a'^2 + \xi a^2 = 2a_0 a - a_1^2 \tag{21}$$

The solution of this equation for the scale factor $a(\eta)$ one can write in the form:

$$a = a_m + \frac{1}{2} a_0 \eta^2 \tag{22}$$

for flat ($\xi = 0$) models (here $a_m = a_1^2/2a_0$),

$$a = a_m + (a_0 - a_m)(1 - \cos\eta) \tag{23}$$

for closed ($\xi = +1$) models (here $a_m = a_0 - \sqrt{(a_0^2 - a_1^2)}$), and

$$a = a_m + (a_0 + a_m)(\cosh\eta - 1) \tag{24}$$

for open ($\xi = -1$) models (here $a_m = \sqrt{(a_0^2 + a_1^2)} - a_0$).

As it is seen, all three models are nonsingular: under $\eta = 0$ the scale factor $a(\eta)$ does not apply to the zero as in standard cosmologycal models, but takes a minimum value a_m.

Let us calculate a value of density ρ_m of nonrelativistic matter at the moment, when scale factor takes a minimum value a_m:

$$\rho_m = \rho_0 \frac{a^3(\eta_0)}{a_m^3}. \tag{25}$$

Here ρ_0 is the value of usual matter density at present moment.

This moment corresponds a time coordinate value η_0. Substituting (22) — (24) into (25) we obtain the expression for ρ_m via ρ_0, η_0 and ratio a_0/a_m.

Let us write down the expressions for $\Omega = (\epsilon_m - \bar{\epsilon})_{\eta=\eta_0}/(c^2\rho)$, and $\lambda = (\bar{\epsilon}/\epsilon_m)_{\eta=\eta_0}$ via η_0 and ratio a_0/a_m.

Here $\rho = 3H_0^2/8\pi k$ is the critical density at the present moment, Ω is the densities parameter, λ is a ratio of "energy density", stipulated by the interaction absolute value, and density of relict radiation energy difference to the density of usual matter energy at the present moment.

From (14), (18) - (20) we obtain

$$\Omega = \left(1 + \xi \frac{a^2}{a'^2}\right)_{\eta=\eta_0} \qquad \lambda = \frac{a_1^2}{2a_0 a(\eta_0)} \tag{26}$$

Substituting (22) — (24) into (26) we obtain the expression for Ω and λ via, η_0 and ratio a_0/a_m.

Expressing from received correlations η_0 and a_0/a_m via Ω and λ and then substituting then received expressions for η_0 and a_0/a_m in (25) , we obtain the following result:

$$\rho_m = \frac{\rho_0}{\lambda^3}\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \lambda(1-\lambda)\frac{(\Omega - 1)}{\Omega}}\right)^3 \tag{27}$$

Result (27) is correct for flat $(\Omega = 1)$, open $(\Omega < 1)$, and closed $(\Omega > 1)$ models.

The received models can be realized, if absolute value of "energy density" (12), stipulated by the interaction, exceeds at present moment the energy density of relict radiation.

The "energy density" (12) , stipulated by interaction, is created basically galaxies accumulations with the mass $m \sim 10^{15} - 10^{16} M_\odot$. This "energy density" is estimated as

$$\frac{12k^2 t_0 \rho_0^2 m}{< v >}.$$

When getting this correlation we put $3k_BT = m < v >^2$, where $< v >$ is an average chaotic velocity of galaxies accumulations, $N = \rho_0/m$, but parameter r_{max} is estimated as $< v > t_0$, where t_0 is the modern age of the Universe (refer to. [10], [11]).

Substituting into the obtained ratio the numerical values of fundamental constants, cosmological time, average density of matter in the Universe at present moment (refer to ,for instance, [14]), we obtain under $<v>/c = 10^{-6}$, $m \sim 10^{15} M_{\odot}$:

$$\epsilon_1 \sim 7 \cdot 10^{-13} erg/cm^3$$

It is approximately the value of relict radiation energy density we have at present moment (refer to.[14]).

Consequently, realization of such situation is possible, when absolute value of "energy density", stipulated by interaction, exeeds the density of relict radiation energy. And it is the case considered in the paper when $\lambda > 0$.

On initial stage of the Universe evolution the density $\rho_m \sim \rho_0/\lambda^3$ is sufficiently great , so usual scenario of the Universe hot models preserves. For a moment when scale factor takes a minimum value and the density of matter is maximum let us introduse the folloing.

If at this moment a temperature does not exceed the temperatures of nucleon - antinucleons couple annihilation, the density of nonrelativistic mater can be evaluated as

$$\frac{m_p 10^{-8} \sigma T^4}{k_B T},$$

where σ is the constant of Stefan - Boltzman, m_p is the mass of proton.

Comparing this expression with ρ_0/λ^3 one can make a conclusion, that minimum value of scale factor will be reach on leptones stage of the Universe evolution when parameter λ is within 10^{-10} - 10^{-12}, and the temperature will be about $10^{10} K^o$ - $10^{12} K^o$.

3. Domain of the theory application.

When concluding the macroscopic Einstein equations we suggest the distant collision to be the prevalent. It is correct if

$$\gamma = \frac{E_p}{E_k} \ll 1,$$

where E_p is the potential energy of two particles interaction, E_k is the kinetic energy of particles.

For the system of electromagnetical interacting particles in a condition of local thermodynamic balance we can put $E_k \sim kT$, $E_p \sim e^2 n^{1/3}$.

So the condition of validity takes the form

$$\gamma = \frac{e^2 n^{1/3}}{kT} \ll 1. \tag{28}$$

Here e, n, T, k is the charge, density, temperature and Boltzman constant correspondingly.

Let us verify (28) in constructing model under $t = t_m$, when the Universe reaches the maximum density. The majority of electromagnetical interacting particles on this stage compose the electron-positron pairs. It's density is compared on the order of value with the one of relict photons and can be estimated as $\sigma T^4/kT = \sigma T^3/k$, where σ is the Stephan-Boltzman constant. Then for parameter γ we have estimation:

$$\gamma = \frac{e^2 \sigma^{1/3}}{k^{4/3}} \sim 10^{-2} \ll 1.$$

Consequently the application condition of the macroscopic Einstein equations in this model is not violated.

References

[1] H. A. Lorentz. Versush einer Theorie der Elektrishen und Optishen Ercheinungen in Bewegten Korpern. Leiden, (1895)

[2] H. A. Lorentz. The Theory of Electrons. Leipzig: Teubner, (1916)

[3] Yu. L. Klimontovich, Kinetic Theory of Electromagnetic Processes [in Russian],Nauka, Moscow (1980)

[4] Shirokov M. F., I. Z. Fisher, Astron. Zh. **39**, 899 (1962) [in Russian]

[5] G. F. R. Ellis. General Relatyvity and Gravitation, ed B. Bertotti, F. de Felici and a¿ Pascolini. Dortrecht: Reidel. (1984)

[6] R. M. Zalaletdinov, Gen Relativ. Gravit. **25**,673 (1993)

[7] A. V. Zakharov, Zh. Eksp. Teor. Fiz. **110**, 3 - 10 (1996) [Sov. Phys. JETP **83**, 1 - 8, (1996)].

[8] A. V. Zakharov, Zh. Eksp. Teor. Fiz. bf 112, 1153 - 1166 (1997) [Sov. Phys. JETP **85**, 627 - 634 (1997)].

[9] A. V. Zakharov. Macroscopic Einstein equations taking into account the electromagnetic interaction of particles of medium.//In : Noveishie problemi teprii polya. Edit: prof. Aminova A.V., Kazan 1998 p.118-124 (in Russian)

[10] G. S. Bisnovatij - Kogan, I. G. Shukhman, Zh. Eksp. Teor. Fiz. **82**, 3 (1982).

[11] A. V. Zakharov, Zh. Eksp. Teor. Fiz. **99**, 769 - 782 (1989). [Sov. Phys. JETP, **69**(3), 437 - 443, (1989)].

[12] N. A. Chernikov, Nauchn. docl. visch. shcol. Fiz. mat. N 1, 168 (1959).

[13] L. D. Landau, E. M. Lifshitz. Teorija polja. Izd-vo "Nauka" M., (1973). (in Russian)

[14] Ja. B. Zeldovich, I. D. Novikov. Stroenie i evoljutzija Vselennoi.
Izd-vo "Nauka" M., (1975).(in Russian)